

## Exercise 28

For the following exercises, use the Intermediate Value Theorem to confirm that the given polynomial has at least one zero within the given interval.

$$f(x) = -2x^3 - x, \text{ between } x = -1 \text{ and } x = 1.$$

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### Solution

Plug  $x = -1$  and  $x = 1$  into the function.

$$f(-1) = -2(-1)^3 - (-1) = 3$$

$$f(1) = -2(1)^3 - (1) = -3$$

Since  $f(x)$  is a polynomial function (a smooth and continuous function),  $f(x)$  has to take on every value between  $-3$  and  $3$  for  $-1 < x < 1$  by the Intermediate Value Theorem. Therefore,  $f(x)$  has a zero between  $x = -1$  and  $x = 1$ .